

# Index

Accuracy. *See Approximation; Error; Measurement*  
Addition, low-stress, 220–26  
Additivity, 13–14, 20, 23–24, 25, 30  
Aids, teaching. *See Manipulative devices*  
Algorithms. *See Low-stress algorithms*  
Angle(s), 29–30, 72–73, 77, 154–55, 164  
Applications  
    of parallax, 141–42  
    in SI, 132–33  
Approximation  
    of area, 169–70  
    in experimental error, 176–79  
    in measurement, 84  
Area, 24  
    additivity property of, 25  
    approximating, by averaging-rectangle method, 169–73  
    comparison of, 26–27, 51, 75–76  
    congruence of, 15  
    determining irregular, 166–68  
    of a rectangle, 51–52  
    sources of error in finding, 106–8  
    spatial representation of, 46  
Area function, 24–27  
Attribute, measurable, 73–77, 88

Behavior, measuring, 44  
Betweenness, on a line, 23  
Bisection of angle, 154–55

Capacity. *See Volume*  
Carpenter's square, 152–55  
Cavalieri principle, 28  
Clinometer, angle, 164, 195–96  
Commensurability, 115

Comparison  
    of angles, 33, 77  
    of areas, 26–27, 51, 75–76  
    of lengths, 36–37, 42, 45, 73  
    of times, 57, 58, 74–75  
    of volumes, 53, 76  
    of weights, 74  
Computation, 17, 124, 218–19. *See also Low-stress algorithms*  
Congruence property  
    of angle measure, 30  
    in area function, 25  
    in higher-dimensional space, 22  
    of length, 20  
Conservation  
    of area, 46–48  
    of energy, 175, 180, 183–84, 185  
    of liquid volume, 76–77  
    of momentum, 175, 180, 183, 185  
    of volume, 47–49, 53, 54  
    of weight, 55  
Constructions. *See Models*  
Coordinate system, for points on a line, 21  
Curriculum. *See also Measurement program*  
    effects of SI on, 134–36  
    place of low-stress algorithms in, 236–37  
    role of estimation in, 87–104 *passim*, 99–100  
Curve, length of, 24, 156–60

Decimal notation, in SI, 128–29  
Derived scales, 31

- Developable surfaces, 160-61
- Distance, 20. *See also* Distance function  
definition of, 21, 158  
on a line, 21  
on surfaces, 156  
between two points, 15, 157-60
- Distance function  
attributes of, as bases for transfer, 20  
compared with area function, 25-27
- Division  
facts, through low-stress addition, 225-26  
of a line into equal parts, 153-54  
low-stress, 233-36
- Error  
accounting for, in measuring, 144, 146  
anticipating sources of, 105, 108,  
109-10, 114, 175-76  
in approximating, 85  
independent, 177-78  
relative, 17  
types of, 176  
using graphs to evaluate, 94-96
- Estimating  
definition of, 89  
developing skill in, 93-94  
problems in, 111-12
- Estimation  
of area, 101  
of length, 100  
of mass, 101-2  
of time, 101  
of volume, 101
- Fractions, in SI, 129. *See also* Rational numbers
- Function  
angle measure, 29-30  
area, 24  
volume, 27-29
- Geoboard, 76
- Geodesics, 156, 157-58, 158-60
- Graph, 73, 94-96
- Histogram, 177
- Homomorphism, 32, 33
- Incommensurability, 26
- Indirect measure, 30, 31
- Individualized instruction, 67-68, 76
- International System of Units. *See* SI
- Length  
additivity property of, 20, 24  
Archimedean property of, 20  
comparison of, 36-37, 42, 73  
congruence property of, 20  
of a curve, 24  
relations, transitivity of, 39-42
- Line  
ability to represent, 38-39  
betweenness on, 23  
distance on, 21  
division of, into equal parts, 153-54
- Linear measurement, unit of, 42
- Low-stress algorithms, 219-39  
addition, 220-26  
division, 233-36  
multiplication, 226-31  
subtraction, 231-33
- Manipulative devices, 187-209 *passim*
- Mass. *See* Weight
- Materials. *See* Manipulative devices
- Mathematics program. *See* Curriculum Measure. *See also* Measurement;
- Measuring
  - definitions of, 18, 88
  - of distance on a line, 21
  - as a function, 19
- Measurement. *See also* Measure;
- Measuring
  - accuracy in, 82
  - anticipating problems in, 114
  - appreciation for, 65
  - as approximation, 84-85
  - comparison in, 61, 62
  - definitions of, 18, 50, 88
  - error in, 16, 144, 146, 175-76
  - as a function, 89
  - importance of, 63-64
  - in modern technology, 1-10 *passim*
  - perception of, 61-62, 68-73
  - principles for effective teaching of, 61-64
  - skills necessary in, 61
  - steps in process of, 108-9
  - of time, 58
  - using the natural environment, 163-74
- Measurement program  
content of, 66-67  
objectives for, in elementary school, 61, 64-66
- Measure systems, 11, 63  
derived, 31  
as a model, 32, 133-34

- Measuring.** *See also Measure; Measurement*  
angles, 29–30  
definition of, 88  
distances on surfaces, 156  
irregular areas, 166–73  
use of error in, 66  
velocity of a projectile, 175–86  
*passim*
- Metric materials, bibliography of, 210–17
- Metric space, 158
- Metric system. *See Measure; Measuring; SI*
- Models  
building, 18  
of measurement process, 106, 120  
of measure systems, 32
- Modular arithmetic, uses of, 127–28
- Multiplication  
low-stress, 226–31  
facts, through low-stress addition, 225
- Nondevelopable surfaces, 161–62
- Nonstandard measures, 77–79
- Nuffield project, 97
- Number line  
as basis for scales, 66  
on instruments and scales, 84
- Optical range finder, 142–44
- Parallax  
activities with, 142–46  
applications of, 138, 141–42  
definition of, 138, 139  
geocentric, 141  
heliocentric, 141–42  
Model T viewer, 144–46
- Percent, in SI, 131
- Perception  
of angles, 72  
of area, 71  
of length, activities for, 68  
of temperature, 70  
of time, 69  
of volume, 71  
of weight, 69
- Perimeter, 22
- Perpendicular bisector, 155
- Photography, to measure velocity, 147–51
- Piaget, 38, 39, 43, 46, 47, 49, 50, 51, 52, 53, 55, 56, 57, 58, 67
- Place value, metric, manipulative aids in teaching, 208–9
- Ratio, in SI, 130–31
- Rational numbers, in SI, 131–32
- Real numbers, in SI, 132
- Referents, 97–98
- Relations, in SI, 130–31
- Relative error, 17
- Scales, derived, 31
- SI (Système International d'Unités)  
advantages of, 125  
base units of, 80, 123  
conventions of usage, 134  
decimal notation in, 128–29  
de-emphasis of computation in, 124  
dominance of base ten in, 126–28  
estimating in, 132  
as a model for mathematical structure, 133–34  
as a system of measurement, 63  
as used to simplify teaching of measurement, 60
- Space  
congruence property in higher dimensional, 22  
measurement in exploration of, 1–2, 81  
metric, 158
- Spatial representation, 37–39, 46–49
- Speed  
definition of, 56  
fundamental to concept of time, 58
- Sphere, volume of, 109–10
- Standard deviation, 176–77, 178
- Standard measures, 79–80. *See also SI*
- Standard referents  
development of, 65  
functions of, 63
- Subtraction, low-stress, 231–33
- Surface area of developables, 156
- Système International d'Unités. *See SI*
- Temperature, 70–71, 197–98
- Tessellations. *See Tiling*
- Tiling, 50–51  
approach to area, 26
- Time  
comparison of, 57, 58, 74–75  
development of concept of, 56  
length of, 56–57  
measurement of, 58  
speed as basis for concept of, 57
- Transfer  
designing instruction for, 12–34  
*passim*  
of estimation skills, 98

- Transfer (*continued*)  
of learning, 12, 33-34  
as primary goal of measure experiences, 12  
of transitivity, 42  
Transitivity, 36, 39-42 *passim*  
of length relations, 39-42  
transfer of, 42  
Trilateration, 166, 168
- Unit(s)  
of area, 26, 51  
of area measurement, 49  
of linear measurement, 42  
in SI, 123  
of volume measurement, 49, 52
- Vectors, addition of, 131  
Vector spaces, in elementary school, 131
- Velocity, 58. *See also* Speed; Time  
measuring, of a projectile, 175-86  
*passim*  
using photography to measure, 147-51
- Volume  
comparison of, 53, 76, 77  
concepts of, 47-49  
difference in liquid and solid, 28  
differentiating length and area, 28  
function, 27-29  
mathematical structure for measure of, 27-29  
models in teaching, 46  
properties of, 27-28  
of a rectangular solid, 52-53  
sources of error in finding, 109-10  
spatial representation of, 46-49  
of a sphere, activity in finding, 109-10
- Weight, 55, 206-7

